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Preface

Uncertainty in structural dynamics

The effects of uncertainty are of growing concern in the design of engineering structures. The fact that the properties of the structure are uncertain implies that there is consequent uncertainty in the dynamic response. Similarly, there is inevitable manufacturing variability: mass-produced items are never identical. Indeed the properties of an individual system will change with time due to environmental conditions, loads, wear, etc.

Uncertainty and variability raise issues concerning safety, reliability, quality of performance, worst-case behaviour and so on, and in turn these issues lead to demands for modelling methods which specifically include uncertainties in the properties of the structure. In the past, factors of safety might be introduced. However, the desire for greater efficiency, improved performance and reduced costs has led to a demand for improved computational methods, especially for high-cost structures. The goal is to apply such methods at the design stage to produce structures which are safe, reliable and have acceptable noise and vibration performance under all environmental and operating conditions which they are expected to encounter, and to produce designs which are robust with respect to manufacturing variability.

Uncertainties in structural properties propagate through the system to give uncertainties in the response: natural frequencies, frequency response functions (FRFs) and so on. One possible approach to quantifying response uncertainties is through Monte Carlo simulation. However, the computational cost of even one structural dynamic response calculation is often high, so that the cost of repeating the analysis many thousands of times, or more, is prohibitive, especially given the fact that practical structures often have very many uncertain parameters. This has stimulated the search for more sophisticated, improved approaches to the problem of uncertainty in structural dynamics.

This Special Issue contains 14 papers which address different aspects of this subject. It grew from a one-day meeting on Uncertainty in Structural Dynamics held at the University of Southampton in 2003.

Broadly, uncertainty in properties is usually described in terms of one of two contrasting views. In probabilistic approaches the properties have assumed statistical distributions and the aim is to predict response statistics. However, quantifying the statistics of the properties is problematical at best, especially in an industrial context. In possibilistic approaches, on the other hand, properties are assumed to lie in certain ranges and no attempt is made to describe any probability distributions within these ranges. However, setting the bounds for the ranges is also

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problematical. In practice, the engineer is likely to make (probably coarse) estimates of input parameter uncertainties based on prior experience and perhaps a very limited number of measurements. This difficulty in quantifying uncertainties in physical properties has prompted some researchers to incorporate uncertainties in a non-parametric manner.

The effects of uncertainty vary with frequency, and this also affects the approach to the analysis. At low frequencies, and when levels of uncertainty are low, the response is typically described by fairly well-defined natural frequencies and the FRFs for the ensemble are spread around that of the deterministic, baseline, or "best-guess" system. It is here that possibilistic and perturbational methods tend to be of most value. As frequency increases, so do uncertainties in the natural frequencies and hence so does the stochastic overlap (the ratio of the standard deviation of the natural frequencies and the expected value of their spacing). When the stochastic overlap is greater than 1 the deterministic "spine" in the ensemble responses becomes lost. The situation is usually ameliorated by the fact that the modal overlap tends to increase with increasing frequency, so that response variability reduces. At high frequencies it appears that natural frequency statistics asymptote to a universal distribution governed by Gaussian Orthogonal Ensemble (GOE) statistics, so that the exact nature of uncertainty in physical properties becomes unimportant if the uncertainties are large enough.

The organisation of the papers in this Special Issue is broadly in terms of, first, possibilistic and then probabilistic approaches and in terms of increasing levels of uncertainty or increasing frequency.

The first ten papers in this Special Issue are broadly concerned with low-frequency structural dynamics. This is the regime in which the natural frequencies or modes of the systems are comparatively well-separated and the quantities of interest are the FRFs, natural frequencies, mode shapes and damping. For such systems the usual approach taken in structural dynamics is modal analysis. Possibilistic approaches are considered in the first four papers.

The paper by Moens and Vandepitte is the first part of *A Fuzzy Finite Element Procedure for the Calculation of Uncertain Frequency-Response Functions of Damped Structures* and is concerned with developing the numerical algorithm for the interval finite element procedure. This is achieved via a set translation of the deterministic modal superposition algorithm within a fuzzy arithmetic framework. As is common, the fuzzy analysis is conducted using interval arithmetic conducted at a series of α-cuts of the uncertain parameters. A summary of the situation for undamped structures is first given, in order to lend clarity to the more complicated situation for damped structures. The well-known interval arithmetic problem of dependency is circumvented to a large extent via the use of an optimisation procedure to calculate the exact modal parameter ranges. These parameter ranges are then used to calculate the total envelope FRFs via the summation of modal envelope FRFs. The method is then extended to consider proportionally damped structures. The real and imaginary FRFs are considered separately through the procedure, until the final stage, where they are combined to give amplitude and phase envelope FRFs.

The paper by De Gersem et al. is the second part of this two-part contribution and it demonstrates the applicability of the methodology via four case studies. The first case study considers a clamped plate structure with uncertain boundary conditions. The interval approach is benchmarked against Monte-Carlo simulations and it is shown that the interval approach introduces conservatism whilst the Monte-Carlo results are dependent upon the adopted sampling strategy. The second case study considers the Garteur benchmark aircraft, designed to evaluate ground vibration test techniques. Three sources of uncertainty are considered: the thickness of a viscoelastic layer, the stiffness of the wing/fuselage connection and the Young's modulus of the wing material. The benefits of eigenvalue interval correction are demonstrated in this case study. The third case study considers a solar panel with stiffened areas to withstand local loads. In the design phase of these panels there is a stiffness to weight issue to be considered and the case study considers the effect of altering the diameters of the stiffened areas upon the dynamic displacements of the panel tips. The interval method is compared with a vertex analysis approach and the conservatism of the interval method away from resonance peaks is noted. Extending the analysis to a fuzzy analysis shows the evolution of tip displacement with increasing diameter of the stiffened areas. The final case study considers the dynamic behaviour of a telescope baffle cover. Six uncertain parameters are identified and their effects on the eigenfrequencies and FRFs of the baffle cover are investigated.

The third paper, by Manson, is concerned with *Calculating Frequency Response Functions for Uncertain Systems using Complex Affine Analysis*. This paper presents an alternative to Interval Analysis for the purpose of propagating uncertainty through a simple model in order to calculate the FRFs of a system. Whilst Interval Analysis is known to generally suffer from overestimation through the inability to account for relationships between variables, this alternative, known as Affine Analysis, is able to account for such dependencies. In this paper, Complex Affine Analysis is developed for the purpose of calculating FRFs of damped structures. The method is illustrated using a simple lumped mass system with uncertain mass, stiffness and damping parameters and compared with Interval and Monte-Carlo approaches. The FRF bounds returned by Complex Affine Analysis are significantly narrower than those returned by Complex Interval Analysis. It is also demonstrated that the affine approach returns a measure of conservatism thus allowing inner and outer bounds on the FRF envelope to be constructed. The paper concludes with a discussion of the role Affine Analysis may play in the analysis of more complex engineering problems.

Following this is *Assessment of Uncertainty on Structural Dynamic Responses with the Short Transformation Method*, by Donders et al. The paper considers an alternative approach to that of Moens and Vandepitte, described earlier in this issue, for obtaining FRFs of damped uncertain structures. It takes as its starting point Hanss' Reduced Transformation Method which is itself an extension of Vertex Analysis. The Transformation Method employs a deterministic approach to assessing the effects of uncertainty thus avoiding the overestimation issue associated with Interval Arithmetic. The authors highlight that the problem with the Reduced Transformation Method is the large number of model runs which are required when there are more than a few uncertain parameters. The Short Transformation Method is proposed to reduce the computational load. This is achieved by identifying the Principal Diagonal of the uncertain parameter hypercube which is simply the diagonal which has the largest contribution to the shape of the FRF envelope. This is ascertained using a heuristic procedure based upon identifying the global envelope of the FRF then identifying its characteristic segments. Once the Principal Diagonal is identified the higher levels of fuzzy membership are only investigated along this diagonal, drastically reducing the number of required model runs. The method is demonstrated using two case studies: a clamped plate and a car front cradle, each subject to three uncertain parameters. The Short Transformation Method and the Reduced Transformation Method are validated against a Monte-Carlo approach using two performance criteria based upon the percentage of Monte-

Carlo samples contained within the global FRF envelopes and a Variance Accounted For measure. Both the Reduced and Short Transformation methods return envelopes which are almost, although not fully, conservative for these non-monotonic case studies. The significant reduction in number of computations in the Short Transformation Method does not significantly affect its performance.

The following paper by Takewaki and Ben-Haim is concerned with *Info-Gap Robust Design with Load and Model Uncertainties*. The motivation for the paper lies with the fact that in the earthquake response of civil structures, the critical excitations will depend on the dynamic properties of the structure and it is therefore necessary to take into account uncertainties in both the loads and the structural model as part of the design process. Because earthquakes are rare and because civil structures are usually unique structures (in some sense), it is argued that a probabilistic approach is suboptimal. The design approach taken in the paper is based on the information-gap model of uncertainty and is motivated by the theorem that maximising design performance will inevitably lead to reduced tolerance to uncertainty—or robustness. The robust design methodology espoused here is based on the idea that one should *satisfy* critical performance criteria while maximising robustness to uncertainty. The paper illustrates a number of important features of the robust design process through a number of illustrative examples: two SDOF systems with differing damping and a 2 storey model shear building. This leads to the new design concept incorporating uncertainties in both the loading and the structural model and this is illustrated on a 6 storey model shear building.

The Special Issue continues with *A Robust Model-Based Test Planning Procedure* by Vinot et al. This paper attempts to answer the question 'how sensitive is my measure of design success to uncertainties in my system representation'? The approach adopted is based on the info-gap method already encountered in the paper by Takewaki and Ben-Haim: i.e. it is an approach which attempts to satisfy critical performance requirements while maximising robustness against uncertainty. The approach is illustrated by designing a vibration test plan for a complex mechanical system including a specification for 'optimal' placement of sensors and actuators. The objective is to identify modal parameters from frequency response functions generated in a base excitation test. The performance figures of merit chosen are the observability and distinguishability of the measured modes. A global mode selection criterion is used to filter out local behaviours which are considered unimportant. The overall procedure is illustrated by application to a payload carried on a satellite.

The next paper is *Uncertainty Identification by the Maximum Likelihood Method*, by Fonseca et al. This is concerned with estimating the variability in model parameters. Given the PDF of a number of response variables, the problem is to determine the probability density function (PDF) of the model parameters. In this case, the authors specify a Gaussian distribution and therefore reduce the problem to finding the mean and covariance information. Inverting the response function directly can lead to problems due to ill-conditioning of the inversion, so the authors adopt an approach whereby the uncertainty in the parameters is quantified by maximising the likelihood of the experimental data. Two common uncertainty propagation methods are considered: a perturbation approach and Monte Carlo. An illustrative example of a cantilever beam is presented where the random variable is the location of a point mass on the beam and the measured responses are natural frequencies. This is validated experimentally. The authors show that the perturbation method is accurate as long as the linearisation of the response variable about the operating point is appropriate. The Monte Carlo approach also gives excellent results, but it is seen that it can be computationally expensive.

The next paper is by Worden et al. on *Some Observations on Uncertainty Propagation Through a Simple Nonlinear System*. The basic premise of the paper is that nonlinear systems present particular problems for uncertainty propagation, because, unlike linear systems, they can bifurcate, i.e. switch between widely divergent behaviours. In order to illustrate the likely problems, one of the simplest possible nonlinear systems is considered—the single degree-of-freedom Duffing oscillator. It is shown that two basic techniques of uncertainty analysis break down when applied to the nonlinear system response in its bifurcating regime. Response surface analysis is considered first and it is argued that the nonlinear behaviour leads to a response surface with fractal properties which make it very difficult to characterise. The second method considered is the First Order Reliability Method or FORM. The FORM analysis breaks down essentially because it proves impossible to compute a gradient on the response surface. The paper concludes with a number of general assertions regarding uncertainty analysis of nonlinear systems and proposes the thesis that problems of this nature in structural dynamics fall into three classes concerned respectively with *quantification, fusion* and *propagation*.

The next paper is by Soize on *A Comprehensive Overview of a Non-Parametric Probabilistic Approach of Model Uncertainties for Predictive Models in Structural Dynamics*. This paper treats both data uncertainties and model uncertainties as issues which must be taken into account to improve the predictive capability of models. It is argued that, although a parametric statistical approach is suitable for data uncertainties, it is not appropriate for model uncertainties as the true experimental system may not be in the span of models reachable by varying the model parameters. For this reason, the author advocates a non-parametric approach to model uncertainties which is based on random matrices. It is shown that such an approach allows one, in principle, to arrive at a better approximation of the experimental system. The main problem in the method is the construction of the PDFs of the matrices of interest. The author describes how this is accomplished using two existing ensembles of random matrices. The analysis allows the calculation of dispersion parameters for the model. The paper describes in some detail how the non-parametric approach can be used to solve the stochastic equations of motion of the dynamical system of interest. The approach is illustrated and validated on a simply-supported beam modelled using finite element (FE) analysis. Confidence bounds are computed for the model frequency response functions.

In *A Consistent Concept for High and Low Frequency Dynamics Based on Stochastic Modal Analysis* Pradlwarter and Schueller consider the energy response of a discrete model of a structure. A deterministic analysis is used to find the global modes of the system as a whole. These are then processed to find the kinetic energy for random, stationary excitation. Subsystem kinetic energies can then be found and are related to input powers by energy influence coefficients. This gives an energy distribution model of the structure which is similar in form to the inverse of an SEA model, but which involves none of the assumptions and approximation inherent in SEA. Furthermore, it is valid for arbitrary distributions of excitation and damping in the system, and for non-conservative coupling. It is noted that the energy influence coefficients are relatively insensitive to uncertainties in the modal properties of the system and the form of the model is valid regardless of the strength of coupling.

The four remaining papers concern uncertainty at higher frequencies, when there are typically large levels of uncertainty. The size of the structure (compared to the shortest wavelength) is typically so large that a full FE model, even were the input data to be known exactly, would be prohibitively large—there are simply too many modes. Energy-based methods now come to the fore.

In *Vibro-Acoustic Analysis of Complex Systems*, Shorter and Langley propose a novel approach to the "mid-frequency" problem of a system comprising subsystems of two quite different characteristics. "Deterministic subsystems" have no uncertainty and typically are long-wavelength, have few modes and are straightforward to model deterministically using FE. "Statistical subsystems" on the other hand have very high levels of uncertainty and are typically short wavelength, have many modes and would typically be modelled using Statistical Energy Analysis (SEA). The analysis of such structures poses substantial difficulties: too large for FEA of the whole structure, yet some subsystems are not amenable to SEA because of their low modal densities. They develop a hybrid approach, in which dynamic stiffness models for the deterministic subsystems are coupled to the statistical subsystems. The latter are described in wave terms, with the response being the sum of a direct field (which might include the effects of any near, well-defined boundary) and a diffuse, reverberant field (which is produced by wave reflections from a distant, random boundary). The subsystems are coupled by "deterministic" boundary degrees of freedom. They show that the conventional wave approach to SEA can be regarded as a special case of the proposed approach, and illustrate its application with an example.

Cotoni et al. consider the issue of variance estimation when using SEA to predict vibrational energy levels, in the paper *Numerical and Experimental Validation of Variance Prediction in the Statistical Energy Analysis of Built-up Systems*. SEA models themselves predict the mean energy levels for an ensemble of structures with random properties. Of course a statistical approach is not complete without some estimate of ensemble variance, together perhaps with confidence limits, etc. The authors present numerical and experimental studies which validate variance estimates given in recent publications. These estimates are found by assuming that the subsystem natural frequency distributions are governed by GOE statistics. Such distributions arise in studies of random matrix theory and are characterised in part by a Rayleigh distribution of natural frequency spacings. Relatively simple expressions for the variance exist, and comparisons are made with numerical simulations and experimental studies in which ensembles of structures with random properties are generated by adding small point masses at random locations to the original structure.

The paper by Wolff and Weaver, *Towards a Diffusion Model of Acoustic Energy Flow in Large Undamped Structures*, concerns diffusive models of energy transport phenomena in large structures at high frequencies. The (slow) evolution of band-limited vibrational energy is assumed to follow a diffusive model. The form of such models and their relation to SEA are discussed. The broader aim is to estimate the parameters of such a diffusive model from short-time direct numerical solutions. The undamped case is often considered for these solutions for the sake of simplicity. There is no need in principle for a formal substructuring procedure. Such an approach allows (in principle) the large-time mean-square (energy) response to be inferred from short-time numerical solutions. The emphasis in the paper is placed on the estimation of the parameters of the diffusive model, and two case studies are presented.

The final paper of this Special Issue, *Ensemble Energy Average and Energy Flow Relationships for Non-stationary Vibrating Systems*, also concerns non-stationary energy modelling at high frequencies. Carcaterra considers the case of a system divided into two subsystems. Expressions for the ensemble average energies are found assuming randomness in the natural frequencies. The averages of a set of random samples are seen to be representative of the single case if the system is "complex enough". The relationships between the energy flow and the (weighted) energy difference are discussed. The evolution of the response in the large-time limit is considered and the damped and undamped cases discussed.

In summary, these 14 papers provide a brief view of some current research work in the area of Uncertainty in Structural Dynamics. There is significant activity in this field and progress is undoubtedly being made. Understanding of the underlying physics is improving and there are clear advances in the development of numerical predictive methods, both possibilistic and probabilistic, that can predict uncertainties in response given uncertainties in input parameters, and with an acceptable computational cost. However, substantial challenges remain and work in this area will no doubt continue in the coming years. It is hoped that this Special Issue will help to inform the reader of current developments and also stimulate further work in this field.

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